Preface

Welcome to Academic English for Mathematics, a complete course for international students preparing to study Mathematics in English at university level!

What's in the course?

Academic English for Mathematics is designed specifically to improve your ability to study Mathematics effectively in English. It is a joint effort of a English for Specific Academic Purposes Instructor and a Mathematician. It is written for international students who are planning to embark on an undergraduate programme of Mathematics and speak English as a foreign language. With this course, you will develop your knowledge of academic and scientific conventions and you will improve your skills in the following areas:

- reading and understanding of Mathematical articles, theorems, proofs, axioms, definitions and word problems in English
- listening to lectures, understanding sign-posting language, main points and improve your note-taking skills
- noticing writing conventions for different audiences and purposes within the same discipline and improving your academic writing skills
- contributing effectively in seminars and discussions
- preparing and giving effective scientific presentations
- improving academic vocabulary and prominent language features
- improving study skills such as planning and note-taking
- improving your critical reading and writing skills with peer-review evaluations.

What's in a unit?

Academic English for Mathematics has twelve (12) units. It has been designed to analyse, sequence and present the target language in a way that accords to foreign language learning needs and a common core Math 101 college syllabus. It is an attempt to match the goals and objectives of a General Mathematics programme and English for Academic Purposes (EAP) featuring formality, paraphrasing, summarising and use of evaluative language. Each unit starts with the learning objectives for that unit. Also, warm up activities are designed so as to ensure that the learner is engaged by the text and is willing to authenticate it by taking interest. At the back of the book there is a glossary with mathematical symbols and audio scripts.

Stimulating activities challenge students, arouse interest, generate discussion, introduce the topic of the unit and encourage rapid progression. The discussion section contains pictures, diagrams and tables contributing to the interest of the reading text.

Reading

The readings although simplified offer undergraduate students of Mathematics a balanced coverage of texts and relevant topics. Learners are exposed to only a few samples of authentic language that maintains the specific features of cohesion and coherence and genre analysis.

To ensure that the texts are accessible to learners at this early stage, texts were written, edited or chosen in accordance to the criteria of length, density of new information and presence of accompanying material such as pictures and diagrams.

Reading comprehension

Each text is followed up by an appropriate reading comprehension task. Students are often asked to read for and identify specific information or show understanding of the general gist, match topics to paragraphs and classify information.

Vocabulary building

Context is seen as vitally important. The book has established a coherent context and is written in support of thematically integrated skills and as such all the skills-building sections are thematically linked. Units and exercises connect in terms of topic and skill development. Exercises, either lexical or grammatical, attempt to engage the learner with the text.

Writing and academic style

This textbook also offers meaningful examples and a variety of techniques for teaching general English for Academic purposes reading and writing skills. Students should build on the functions of language such as coherence and cohesion, exemplifying, cause and effect, summarising, peer-reviewing and evaluating. Students are given guidance to develop these important skills in confidence.

Presentation and discussion skills

Students are guided with regards to presentation structure, presentation slide design and delivery. They are also encouraged to discuss topics that are relevant to each unit and help them practice their conversational skills as well as encourage them to authenticate the topic presented later on.

I hope this book comes up to your expectations. I wish you every success and hope you enjoy your time learning English for Mathematics.

Kallia Katsampoxaki-Hodgetts

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3 Exponents and Exponential Functions

Themes
Exponents and exponential functions; exponential decay and growth; geometric progressions and scientific notation

Academic vocabulary
Definitions;
Writing the product as a monomial in a standard form
Numbers of Science:
Conversions from Metric system to English
Metric prefixes for powers of 10
Comparing objects of widely different sizes: orders of magnitude

Academic writing and style
Introduction to paraphrasing;
Expressing cause and effect as a paraphrasing tool

Discussion

Task 1 Can you think of examples of "growth" and "decay" in everyday life?

For example, the worlds' population continues and will continue to "grow" whilst atmospheric pressure "decays" as we climb higher up a mountain.

Task 2 Have you ever heard of the number "*e*"? How would you describe in your own words your notion of this special number?

Reading

Exponents and exponential functions; exponential growth and decay, the natural exponential function, geometric progressions, scientific notation

Exponential functions are a valuable tool to various scientific fields. In this unit, we will capture a glimpse of their essence, along with their growth and decay as well as basic exponential properties.

YEAR	AMOUNT IN €	AMOUNT OF INCREASE IN €
2010	100	-
2011	$100(1.027) = 102.7$	2.7
2012	$100(1.027)^2 = 105.4$	2.7
2013	$100(1.027)^3 = 108.3$	2.9
2014	$100(1.027)^4 = 111.2$	2.9
2015	$100(1.027)^5 = 114.2$	3
2016	$100(1.027)^6 = 117.3$	3.1
2017	$100(1.027)^7 = 120.5$	3.2
2018	$100(1.027)^8 = 123.7$	3.2

2019	$100(1.027)^9 = 127$	3.3
2020	$100(1.027)^{10} = 130.5$	3.5

Table 1 Matrix displaying the increase of a capital stock of 100€ that was invested in 2010 with a yearly interest rate of 2.7%.

The phenomenon of **compound interest** is a vivid example of what mathematicians call an **exponential function** and can be described by a function of the form,

$$y = ca^x$$

where c equals the initial capital stock and a is equal to the unit incremented by the interest rate in decimal format.

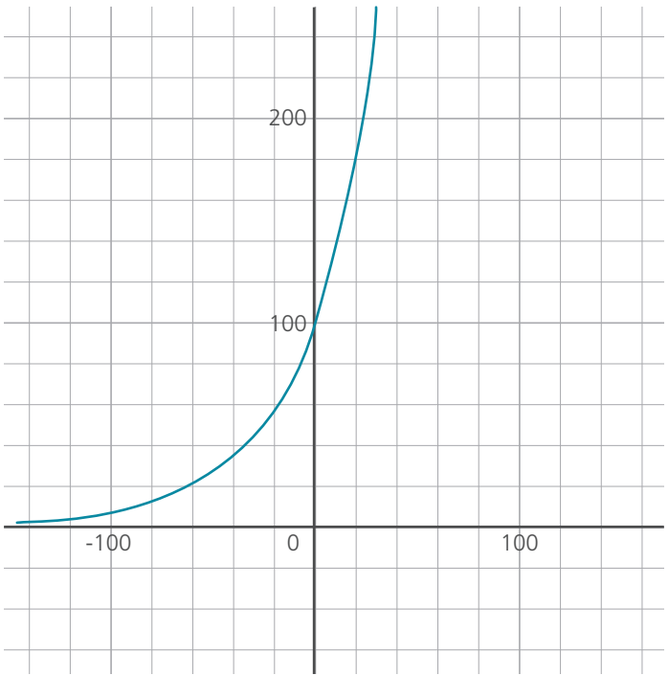


Figure 3.1 Plot of the function $y = ca^x$, where $c = 100$ and $a = 1.027$. What do you notice in relevance with this plot?

In a stricter mathematical frame, an exponential function is a function of the form

$$y = a^x$$

where $a > 0$. The variable x is the exponent. More generally, we often use functions of the form

$$y = a^{x+c}$$

where c is a constant. This can be rewritten as

$$y = ba^{x}$$

where $b = a^{c}$ (derived from an exponential property included in the table below).

If $a, b > 0$ then for $x, y \in \mathbb{R}$ the following statements hold:

$a^{x}a^{y} = a^{x+y}$
$a^{x}b^{x} = (ab)^{x}$
$(a^{x})^{y} = (a^{y})^{x} = a^{xy}$
$\frac{a^{x}}{a^{y}} = a^{x-y}$
$\frac{a^{x}}{b^{x}} = \left(\frac{a}{b}\right)^{x}$

Exponential functions are functions of a real variable and can be distinguished from others, due to the fact that the growth rate of such a function; i.e. its derivative, is directly proportional to the value of the function. Although exponential functions might resemble functions of the form $y = x^{a}$, they are quite different. Their difference lies on the notion that the base of exponential functions is a fixed number and the power is a variable whereas on the latter it is the other way around.

For computational reasons and since replacing the base of an exponential function barely makes a difference in the appearance of a constant factor, we are going to to study a particular function called **"the natural exponential function"**, or simply, **"the exponential function"**.

6 Introduction to Statistics

Themes
Data, sample population, numerical descriptors, rational equations and functions; descriptive and inferential statistics, statistical significance; standard deviation, coefficient of variation, mean, median; graphs

Vocabulary
Statistics: Definitions
Word formation and use-in-context

Language
The use of gerund and infinitive in Mathematics

Presentation/Writing
Reporting graphs and charts

Writing
Writing a report following a chart or graph
Plagiarism, citations and references; why and how we use them

Discussion

Task 1 Look at the following picture and try to describe what it represents, based on your own knowledge of Probability Theory. Does this picture ring a "bell"?

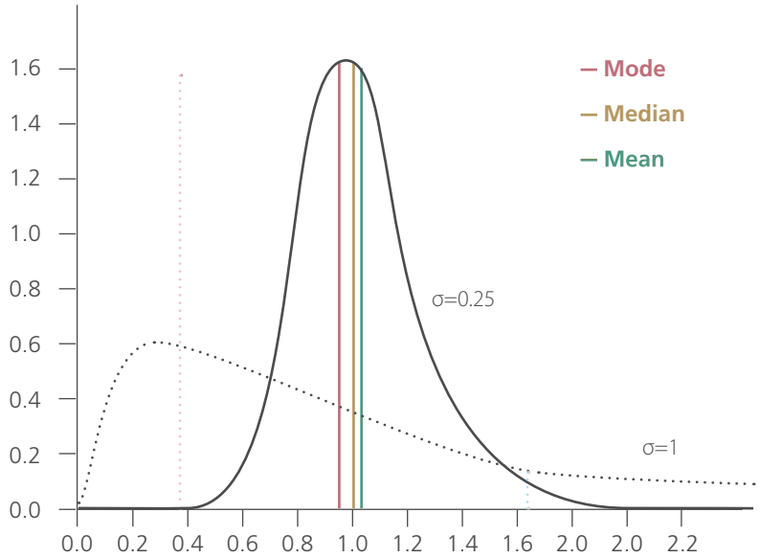


Figure 6.1 Visual representation of a normal distribution curve in a line chart indicating mode, median and mean (always in the center) values, where the *y*-axis denotes the value of the probability density function on different values of *x*.

Task 2 Can you tell the difference between a median, a mode and a mean value?

Reading

Introduction to Statistics

The goal of Statistics is to gain understanding of data. Statistics is a branch of mathematics handling the collection, analysis, organisation and presentation of mass data. Data is, perhaps, the most essential part of this branch, since without it, scientific applications can have little applicability. A dataset is created when related data are collected in one place (Fig. 6.2). In order to make use of it, data is organised into cases and/or random variables, whose values cannot

be predicted in advance. These variables describe the characteristics of a population as phenomena to be examined. Frequently, the goal is to quantify uncertainty while drawing conclusions about the "population", while assessing the strength and weaknesses of these conclusions and evaluating their likelihood to error.

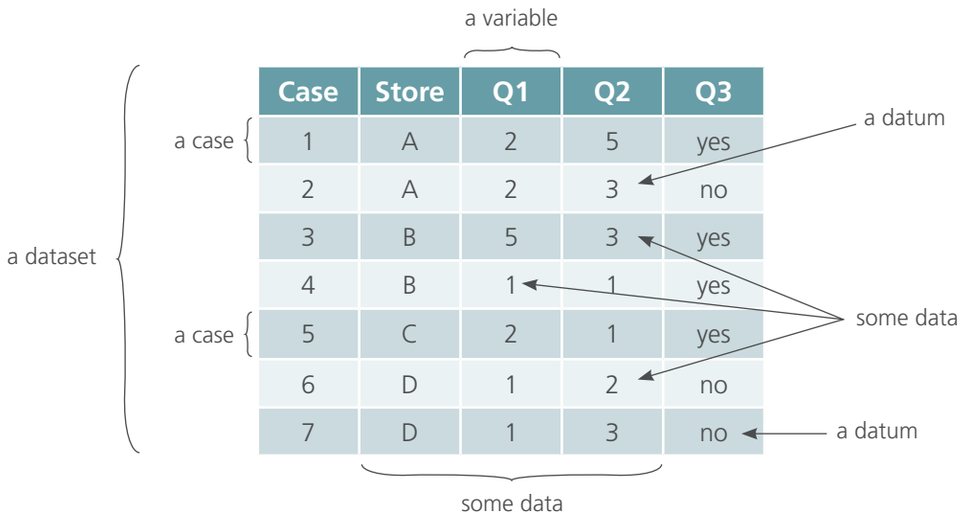


Figure 6.2 Summary of key introductory terms in statistics. From: Landers R.N. (2013) A step-by-step introduction to Statistics. SAGE.

The branch of Statistics concerned with making such inferences about a population of data is known as inferential statistics. Yet, Statistics can be merely descriptive in nature as data analysts can focus only on summarising information. For example, suppose we are interested in the value of a stock share. The characteristics of corresponding values could include either its value per day or its previous value and whether it affects the current one. Each characteristic is perceived as a variable that contains observations or measurements, one for each share.

Although the desired practice would be to compile data of an entire population (i.e. census), it is not always feasible. When that situation occurs, statisticians choose to work with a subset of the population, called a sample. Inferential Statistics go beyond the descriptive calculation of averages, measures of variation and percentiles. In

fact, they include methods that are based on Probability Theory. These methods include point estimation, interval estimation and hypothesis testing.

Descriptive and Inferential Statistics analysis can be interrelated as the former can reveal features that may lead to the selection of the appropriate inferential method. In fact, data analysts formulate the research problem and define the population and the sample before collecting the data. Following descriptive data analysis, they also use appropriate statistical methods to solve a research problem. Once sampling errors and response bias have been eliminated and a representative sample of the population is collected through special observational studies, statisticians summarise the sample data using a number of tools, such as numerical descriptors, diagrams and plots. In order to ensure that conclusions can be safely extended from the drawn sample to the population as a whole without any bias, the use of estimators that validate the true tendency of the sample is essential. Commonly used estimators entail the mode, sample median, sample mean $\overline{X}$, sample variance s^2, sample standard deviation s, sample correlation coefficient r and sample covariance $cov(X, Y, \ldots)$.

Given the statistical significance of presented data is evident and the hypothesis is verified, graphs and schemes can illustrate the relative frequency distribution by providing the percentage of each trend or determining the quantitative variables (i.e. numerical measurements) or qualitative variables on categorical data. Variables can also be described according to the scale on which they are defined. As such, scales can be either nominal or ordinal (e.g. strength of opinion), interval scale (e.g. Celsius degrees for temperature) or ratio scale (e.g. the height of a person). Nominal data (i.e. marital status) tend to be coded by assigning numbers (1 for married, 2 for single, 3 for divorced, 4 for widowed) to each of their categories so that the conversion of categorical data to numerical data is feasible. (Fig. 6.3)

8 Properties of Triangles

Themes
Triangles; main and secondary elements of a triangle, types of triangles by lengths of sides, classification according to internal angles, the Pythagorean Theorem, the concepts of congruence and similarity

Vocabulary
Definitions; use-in-context: types of triangles
Writing a two-column proof
Making comparisons

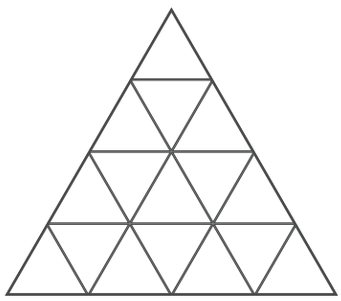
Writing
Passive voice

Presentation
Opening/closing phrases and transitions

Discussion

Task 1 In nature there are plenty of examples expressing a triangular behaviour. Can you name a few?

Task 2 How many triangles can you spot in the following picture?



Reading

Classification of triangles and properties

In the previous unit, we talked about basic concepts and terms of Euclidean Geometry that include points, lines and angles. In this chapter, we are going to have an overview of one of the fundamental shapes that characterise our world; the shape of **triangles**.

Figure 8.1 The Louvre Pyramid, built in 1998 by Chinese-American architect I. M. Pei out of glass and steel. It is located at the main entrance of the museum, underneath the Napoleon Court and is claimed to have 666 triangular glass-panels.

A **triangle** is a **closed shape** that is the union of exactly three line segments which intersect only at their endpoints. The line segments are called **sides** of the triangle. By using the term "closed", we mean that if we start drawing a straight line at any point on the figure and trace it along the remaining two sides, we will come across the starting point again.

A triangle $\triangle ABC$ or simply ABC consists of **three vertices** *A, B, C*; **three sides** *AB, BC, CA* and **three angles** $\angle BAC, \angle ABC, \angle BCA$. For convenience, mathematicians use lowercase letters to represent the length $\overline{AB}, \overline{BC}, \overline{CA}$ of the sides such as *a*, *b*, *c,* respectively, and capital letters to symbolise angles, such as $\hat{A}, \hat{B}, \hat{C}$. The sides and angles of a triangle are considered to be its **main elements**.

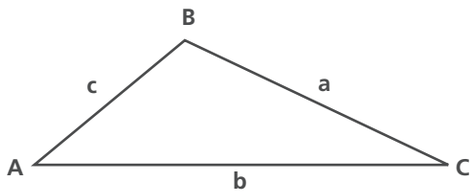


Figure 8.2 The shape of a triangle.

We often say that **a side**, for instance *AB*, **is included** between the angles $\angle BAC, \angle ABC$ if the endpoints of that segment are the vertices of two angles e.g. in figure 8.2, *CA* is included between $\angle BCA, \angle CAB$.

Similarly, we say that an angle is **included** between those sides of a triangle that are contained in its rays. For example, using the same triangle as before, we see that the angle $\angle BAC$ is included between $\overline{AB}, \overline{AC}$.

Note that for each side of a triangle there is a vertex that does not correspond to an endpoint of that particular side. For example, in

triangle $\triangle ABC$, B is not an endpoint of CA. In this case, we shall say that $\angle ABC$ **is the angle opposite side** CA or that CA is the **side opposite** $\angle ABC$. For instance, in triangle $\triangle ABC$, $\angle BCA$ is the angle opposite AB.

Secondary elements of a triangle

- A **median** of a triangle is the line segment that unites a vertex with the midpoint of the opposite side.

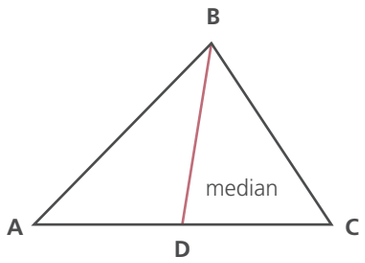


Figure 8.3 The line segment BD is the median that corresponds to the side AC of the triangle $\triangle ABC$.

- An **angle bisector** of a triangle is the line segment through a vertex that divides the angle in two equal parts.

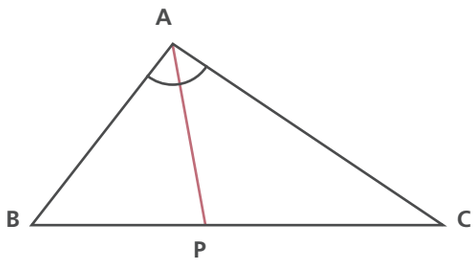


Figure 8.4 The line segment AP is the bisector of the angle $\angle BAC$ if $\angle BAP, \angle PAC$ are equal.

- An **altitude** of a triangle is the perpendicular line segment that is drawn from a vertex towards the straight line of the opposite side. The opposite side is called the **base of the altitude**, whilst the point at which the altitude intersects the base is called **the foot of the altitude**.

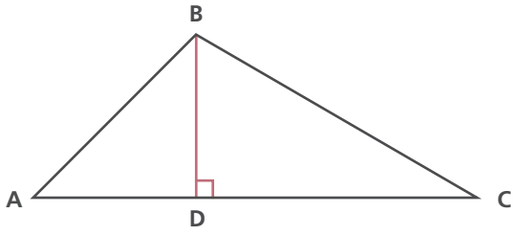


Figure 8.5 The line segment *BD* represents the altitude of the triangle through vertex *B*. The endpoint *D* is called the projection of *B* over the side *AC*, or even trace of the perpendicular over *B* to the side *AC*.

The perimeter of the triangle is defined as the sum of a triangle's sides $a+b+c$.

A very important property of triangles in Euclidean Geometry is the following:

The sum of a triangle's interior angles measures to 180°.

Triangle classification

Classifying triangles according to sides

- A **scalene** triangle is a triangle whose sides are non-congruent in pairs.

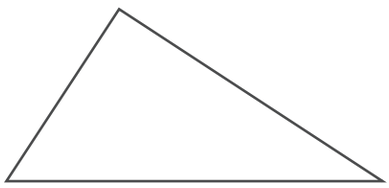


Figure 8.6 The graphical representation of a scalene triangle.

Glossary

abridge		(v.) to shorten
absorb		(v.) to take in or soak up
abstract		(adj.) pertaining to abstraction i.e. the process of extracting the essence of a mathematical concept, removing any dependence on real world objects and generalising it so that it has wider applications
acceleration		(n.) the rate of change of velocity of an object with respect to time
accept		(v.) regard favourably, approve as true or valid
accurate		(adj.) free from error or defect; consistent with a standard, rule or model
acute		(adj.) intense, severe // ~ angle: an angle whose degree measure is greater than zero and less than 90° // ~ triangle: a triangle whose interior angles are all measuring less than 90° // ~ trapezoid: a trapezoid that has two adjacent acute angles on its lower base edge
addend		(n.) any of a group of numbers or terms added together to form a sum
adder		(n.) the arithmetic component computed during addition of positive numbers
addition		(n.) the process of uniting two or more numbers into one sum, represented by the operating symbol "+"
additive		(adj.) a number added to a product // ~ property of equality: a property that allows one to add the same quantity to both sides of an equation
adequate		(adj.) sufficient for a specific need
adherent (point)		(adj.) an adherent point of a subset A of a topological space X, is a point x in X such that every open set containing x contains at least one point of A

adjacent		(adj.) lying next to each other // ~ angles: a pair of angles that are next to each other sharing a line // ~ sides: sides that are next to each other sharing a common vertex // ~ arcs: arcs that are next to each other on the circumference of a circle
adjoined		(adj.) to be in contact or in connection with
adjoint (equation)		(adj.) a linear differential equation, usually derived from its primal equation using integration by parts
adjunction		(n.) a possible relationship between two functors
affinity		(n.) an affine transformation preserving collinearity
affirm		(v.) to validate, confirm
affix		(n.) a group of letters that is added either to the beginning or end of a word
aggregate		(n.) an amount or score made up of several smaller amounts or scores added together
algebraic		(adj.) pertaining to algebra
alignment		(n.) an arrangement in a straight line
alternate (angles)		(adj.) the four pairs of angles that have distinct vertex points, lie on opposite sides of the transversal and both angles are interior or both are exterior
alternative		(adj.) one of two or more possibilities of choices, options, propositions, or courses of action