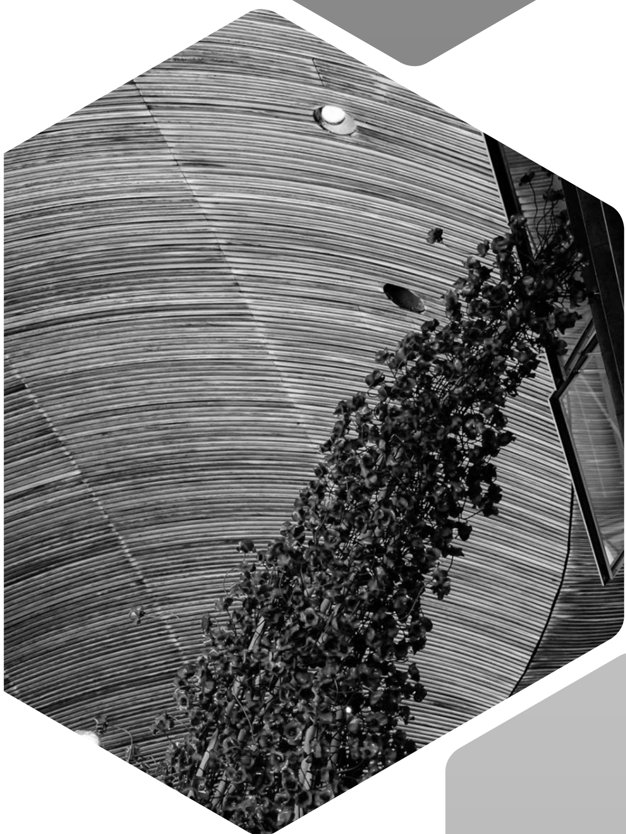
1 Real Numbers



Themes
Properties of real numbers; whole numbers, natural numbers, integers, fractions, rational numbers, square roots, irrational numbers

Vocabulary
Definitions of types of numbers; properties of real numbers, derivatives
Common procedures followed by mathematicians: factoring, using reciprocals/ division, collecting like items, using average, converting to decimal or fractional notation

Writing
Paragraph structure, cohesion and coherence

Note-taking
Identifying main points in a lecture: “Imaginary and complex numbers” contents

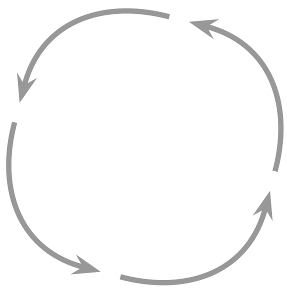
Discussion

Task 1 Look at the pictures and consider what they have in common.

Figure 1.1 Thermometre depicting really low temperatures.

Figure 1.2 A sign marks Badwater Basin lowest point in North America 282 feet below sea level.

Task 2 In pairs, discuss which sets of numbers you were first taught and try to arrange them in a circle that represents their order.



Reading

Properties of real numbers; whole numbers, natural numbers, integers, fractions, rational numbers, square roots, irrational numbers

Our current mathematical mindset has evolved since early humans felt the need to express themselves through numbers. This occurred quite naturally since they reached a point where they would have to count in order to obtain a better level of communication. In fact, they started to count using their fingers as an indicator. As a result, they discovered that:

I	II	III	IIII	…
↓	↓	↓	↓	
1	2	3	4	…

which signalled the beginning of what we now call **natural numbers**.

In mathematics, natural numbers are used mainly for counting and they consist of an infinite ordered set of numbers that can be placed along an axis (Fig. 1.3). Natural numbers are **positive** and **discrete** which means that they can be placed on the number line in consecutive order, with a fixed gap between a number and the next one. It is also customary to consider 0 as a natural number. For the sake of avoiding ambiguation about whether zero is included or not in this set, mathematicians use the superscript "*" to distinguish the two cases. Accordingly,

$\mathbb{N}^* = \{1, 2, 3, \ldots\}$

$\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$ ← Also, known as whole numbers.

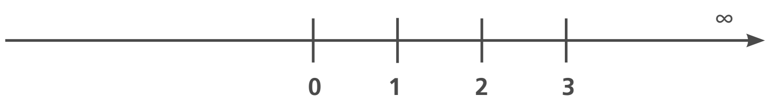


Figure 1.3 The figure above displays the number axis or number line for natural numbers.

Consequently, natural numbers are the basis from which many other number sets can be built by extension, the first of which are **integers**. Integers are whole numbers including zero, the positive natural numbers and their **additive inverses** (also known as the **negative integers**). The set of integers is symbolised as a blackboard bold $\mathbb{Z}$ and can be represented on the number line as follows:

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ ← Ordered and discrete set

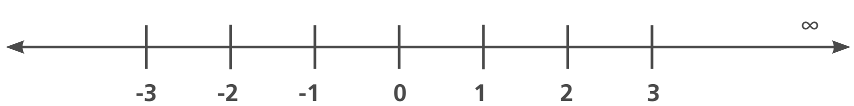


Figure 1.4 A representation of the integers on the number line.

Following integers, a rational number is any number that can be written as a ratio or fraction over of two integers, consisting of the **numerator** and the **denominator** (non-zero). Every integer can be phrased as a rational number, since the denominator may be chosen as the number 1. The **field of rationals** is denoted by the blackboard bold $\mathbb{Q}$. Note that the set of rationals has the property that between any two rationals, there sits another one and, therefore, infinitely many other ones.

Any number that is not rational is called **irrational**. In other words, the set of irrational numbers is $\mathbb{R}\backslash\mathbb{Q}$. This symbol is read as *R* minus *Q*, also known as the set of real numbers minus the set of rational numbers. Typically known irrational numbers are the square roots of natural numbers (excluding perfect squares), the ratio π of a circle's circumference to its diameter, the golden ratio φ, Euler's number *e* and certain logarithms.

Completing the circle of numbers, there appears the infinite set of **real numbers** (Fig. 1.6). Real numbers include all natural numbers, integers, rational and irrational number sets. They can be positive, negative or equal to zero. It is helpful to think of them as **continuous**, infinite set of points along the number line (widely known as the **real line**).

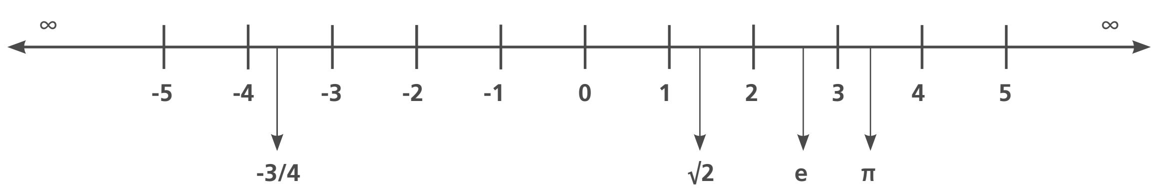


Figure 1.5 Visual representation of the number line containing all real numbers.

They may be expressed by decimal representations that have an infinite sequence of digits to the right of the decimal point. A typical example of their representation is 345.6789321…

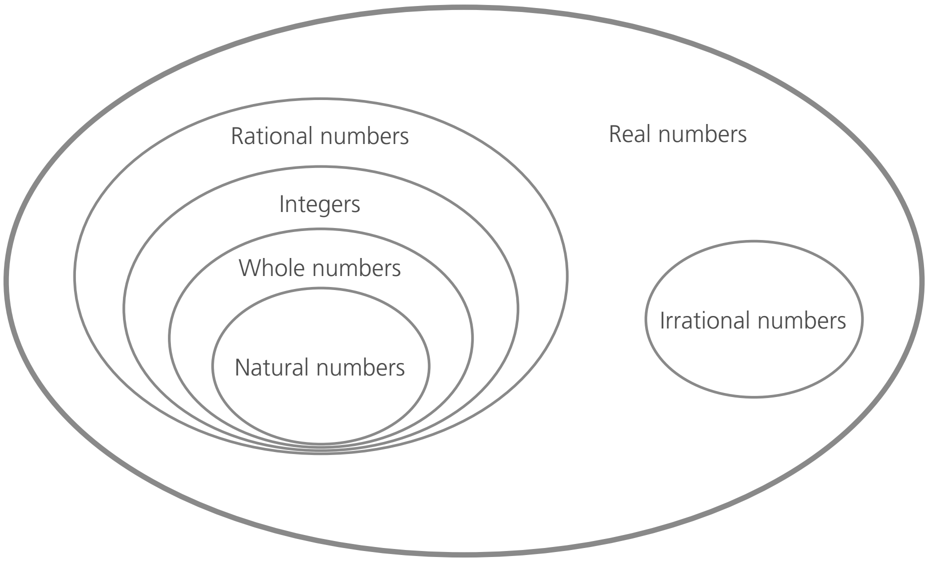


Figure 1.6 A Venn type of diagram representing all types of numbers and their set inclusions.

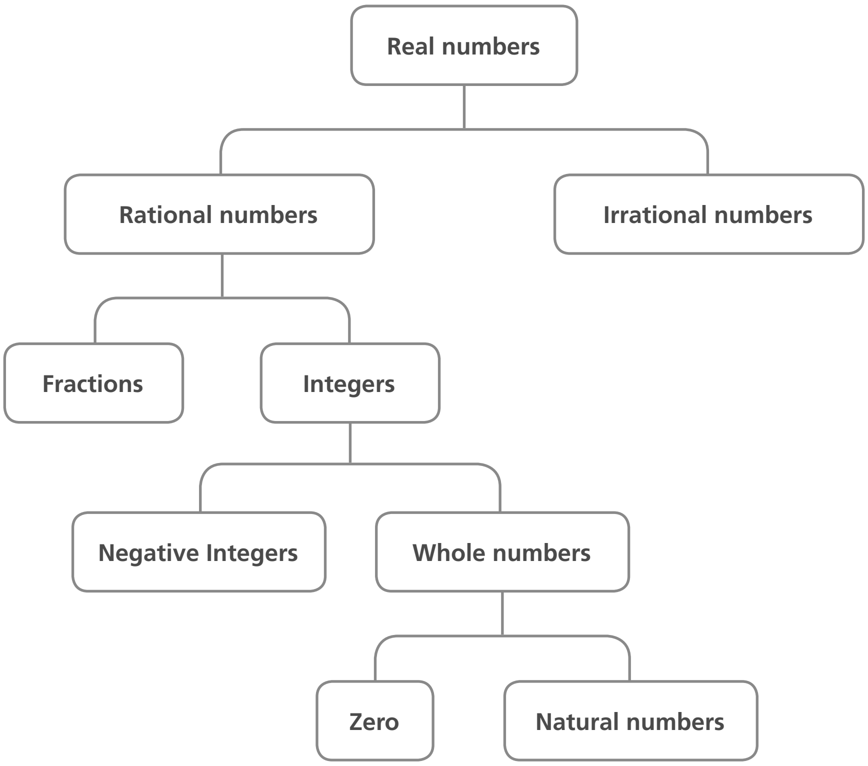


Figure 1.7 A hierarchical diagram representing all types of numbers and their inclusions.

Reading comprehension

Task 3 Read the text and answer the questions:

1 Real numbers are values that represent a quantity along an infinite line. **[Real numbers are non finite points that represent a quality along the real line that is not bounded.]**

2 It is suggested that -5 is an integer but not a real number. **[Minus 5 is both an integer and a real number. The set of real numbers contains the field of integers.]**

3 The square root of two, in symbols √2, is an irrational number, thus a real one. **[The statement is true in this case. However i√2 is a complex number and not a real one so one should be careful about making such assumptions.]**

4 Not all real numbers can be expressed by a possibly infinite decimal representation such as 13,456… **[F Even natural numbers can be represented in the decimal form (e.g. 1,0000….)]**

5 A real number can be rational, irrational, positive, negative or zero. **[A real number can be all the above but not the same time.]**

6 The ellipsis at the end of the decimal number indicates that there might be more numbers. **[The ellipsis indicates that there are infinitely many numbers.]**

Vocabulary

Task 4 Match the definitions with the correct type of number:

rational irrational integer fraction whole numbers

[irrational]
1 Any number that cannot be expressed as a fraction for any integers *p* and *q*. Irrational numbers have decimal expansions that neither terminate nor become periodic.

[whole numbers]
2 Any positive numbers, including zero, without any decimal or fractional parts.

[rational]
3 Any number that can be expressed as the quotient or fraction *p/q* of two integers, a numerator *p* and a non-zero denominator *q*. Since q may be equal to 1, every integer is in the same class.

[fractions]
4 Any number representing a part of a whole or any number of equal parts.

[integers]
5 Any number that can be written without a fractional component.

[*_Note_ that none of these statements should be followed as the strict definition of each set]